Applications of normal distribution

S2CID 259689086. Further information: Interval estimation and Coverage probability For the normal distribution, the values less than one standard deviation from the mean account for 68.27% of the set; while two standard deviations from the mean account for 95.45%; and three standard deviations account for 99.73%. Either conjugate or improper prior distributions may be placed on the unknown variables. Centered at the mean μ= median = mode. These are computed by the numerical method of ray-tracing.[41] The probability density, cumulative distribution, and inverse cumulative distribution of any function of one or more independent or correlated normal variables can be computed with the numerical method of ray-tracing[41] (Matlab code). Probability and Mathematical Statistics. Maxwell, James Clerk (1860). Compute the probability of getting a z-score between –1.37 and 1.68. Therefore, physical quantities that are expected to be the sum of many independent processes, such as measurement errors, often have distributions that are nearly normal.[6] Moreover, Gaussian distributions have some unique properties that are valuable in analytic studies. {\displaystyle s^{2}={\frac {n}{n-1}}{\hat {\sigma }}^{2}={\frac {1}{n-1}}\sum _{i=1}^{n}(x_{i}-{\overline {x}})^{2}.} The difference between s 2 {\textstyle s^{2}} and σ ^ 2 {\displaystyle \textstyle {\hat {\sigma }}^{2}} becomes negligibly small for large n's. "The Hitchhiker's Guide to the Galaxy offers this definition of the word "Infinite." Infinite: Bigger than the biggest thing ever and then some. {\displaystyle \varphi (z)={\frac {e^{\frac {-z^{2}}{2}}}{\sqrt {2\pi }}}\,.} The variable z {\displaystyle z} has a mean of 0 and a variance and standard deviation of 1. Excel: z = NORM.S.INV(0.25) = -0.6745. ^ Jordan, Michael I. 189) ^ Gauss (1809, section 177) ^ Stigler (1986, p. 110) ^ Bernardo & Smith (2000, p. The estimator s 2 {\textstyle s^{2}} differs from σ ^ 2 {\displaystyle \textstyle {\hat {\sigma }}^{2}} by having (n − 1) instead of n in the denominator (the so-called Bessel's correction): s 2 = n n − 1 σ ^ 2 = 1 n − 1 ∑ i = 1 n (x i − x ¯) 2 . Within Stein's method the Stein operator and class of a random variable X ∼ N (μ , σ 2) {\textstyle X\sim {\mathcal {N}}(\mu ,\sigma ^{2})} are A f (x) = σ 2 f ′ (x) − (x − μ) f (x) {\textstyle {\mathcal {A}}f(x)=\sigma ^{2}f'(x)-(x-\mu)f(x)} and F {\textstyle {\mathcal {F}}} the class of all absolutely continuous functions f : R → R such that E [| f ′ (X) |] < ∞ {\textstyle f:\mathbb {R} \to \mathbb {R} {\mbox{ such that }}\mathbb {E} [|f'(X)|] 1). This number serves as a scaling parameter on the variance, making it possible to control the overall variance of the mean relative to the actual variance parameter. In a bell-shaped distribution with mean μ and standard deviation σ, Approximately 68% of the observations fall within one standard deviation (σ) of the mean μ. ISBN 978-0-8247-5402-0. (1994). Here are two examples of histograms with their corresponding quantile plots. This formulation arises because for a bivariate normal random vector (X, Y) the squared norm X2 + Y2 will have the chi-squared distribution with two degrees of freedom, which is an easily generated exponential random variable corresponding to the quantity −2 ln(U) in these equations; and the angle is distributed uniformly around the circle, chosen by the random variable V. First, draw a bell-shaped distribution and identify the two points on the number line. The central limit theorem also implies that certain distributions can be approximated by the normal distribution, for example: The binomial distribution B (n , p) {\textstyle B(n,p)} is approximately normal with mean n p {\textstyle np} and variance n p (1 − p) {\textstyle np(1-p)} for large n {\displaystyle n} and for p {\displaystyle p} not too close to 0 or 1. Compute the z-score that corresponds to the 25th percentile. It is typically the case that such approximations are less accurate in the tails of the distribution. JSTOR 2236741. Once you find the phrase then match up to what sign you would use and then use the table to walk you through using Excel or the calculator. Whether these approximations are sufficiently accurate depends on the purpose for which they are needed, and the rate of convergence to the normal distribution. Zelen & Severo (1964) give the approximation for Φ(x) for x > 0 with the absolute error |ε(x)| < 7.5·10−8 (algorithm 26.2.17): Φ (x) = 1 − φ (x) (b 1 t + b 2 t 2 + b 3 t 3 + b 4 t 4 + b 5 t 5) + ε (x) , t = 1 1 + b 0 x , {\displaystyle \Phi (x)=1-\varphi (x)\left(b_{1}t+b_{2}t^{2}+b_{3}t^{3}+b_{4}t^{4}+b_{5}t^{5}\right)+\varepsilon (x),\qquad t={\frac {1}{1+b_{0}x}},} where ϕ(x) is the standard normal probability density function, and b0 = 0.2316419, b1 = 0.319381530, b2 = −0.356563782, b3 = 1.781477937, b4 = −1.821255978, b5 = 1.330274429. doi:10.18637/jss.v005.i08. Mathematics for Physical Science and Engineering. (1996). With technology, you no longer have to standardize first, we can just find P(X ≥ 69.5). ^ Jorge, Nocedal; Stephan, J. doi:10.2307/2331536. TI-84: Press [2nd] [DISTR]. Many test statistics, scores, and estimators encountered in practice contain sums of certain random variables in them, and even more estimators can be represented as sums of random variables through the use of influence functions. Take the complement 1 – 0.98 = 0.02, then split this area between both tails. Gaussian processes are the normally distributed stochastic processes. ISBN 978-0-02-914673-6. The Empirical rule only gives an approximate value though. Pearson, Karl (1901). The Joy of Finite Mathematics. Comparison of probability density functions, p (k) {\textstyle p(k)} for the sum of n {\displaystyle n} fair 6-sided dice to show their convergence to a normal distribution with increasing n a {\textstyle na} , in accordance to the central limit theorem. "A fast normal random number generator" (PDF). Natural Inheritance (PDF). The cumulant generating function is the logarithm of the moment generating function, namely g (t) = ln M (t) = μ t + 1 2 σ 2 t 2 . Cumulative distribution functionNotation N (μ , σ 2) {\displaystyle {\mathcal {N}}(\mu ,\sigma ^{2})} Parameters μ ∈ R {\displaystyle \mu \in \mathbb {R} } = mean (location) σ 2 ∈ R > 0 {\displaystyle \sigma ^{2}\in \mathbb {R} _{>0}} = variance (squared scale)Support x ∈ R {\displaystyle x\in \mathbb {R} } PDF 1 2 π σ 2 e − (x − μ) 2 2 σ 2 {\displaystyle {\frac {1}{\sqrt {2\pi \sigma ^{2}}}}e^{-{\frac {(x-\mu)^{2}}{2\sigma ^{2}}}}} CDF Φ (x − μ σ) = 1 2 [1 + erf (x − μ σ 2)] {\displaystyle \Phi \left({\frac {x-\mu }{\sigma }}\right)={\frac {1}{2}}\left[1+\operatorname {erf} \left({\frac {x-\mu }{\sigma {\sqrt {2}}}}\right)\right]} Quantile μ + σ 2 erf − 1 (2 p − 1) {\displaystyle \mu +\sigma {\sqrt {2}}\operatorname {erf} ^{-1}(2p-1)} Mean μ {\displaystyle \mu } Median μ {\displaystyle \mu } Mode μ {\displaystyle \mu } Variance σ 2 {\displaystyle \sigma ^{2}} MAD σ 2 erf − 1 (1 / 2) {\displaystyle \sigma {\sqrt {2}}\,\operatorname {erf} ^{-1}(1/2)} AAD σ 2 / π {\textstyle \sigma {\sqrt {2/\pi }}} Skewness 0 {\displaystyle 0} Excess kurtosis 0 {\displaystyle 0} Entropy 1 2 log (2 π e σ 2) {\textstyle {\tfrac {1}{2}}\log(2\pi e\sigma ^{2})} MGF exp (μ t + σ 2 t 2 / 2) {\displaystyle \exp(\mu t+\sigma ^{2}t^{2}/2)} CF exp (i μ t − σ 2 t 2 / 2) {\displaystyle \exp(i\mu t-\sigma ^{2}t^{2}/2)} Fisher information I (μ , σ) = (1 / σ 2 0 0 2 / σ 2) {\displaystyle {\mathcal {I}}(\mu ,\sigma)={\begin{pmatrix}1/\sigma ^{2}&0\\0&2/\sigma ^{2}\end{pmatrix}}} I (μ , σ 2) = (1 / σ 2 0 0 1 / (2 σ 4)) {\displaystyle {\mathcal {I}}(\mu ,\sigma ^{2})={\begin{pmatrix}1/\sigma ^{2}&0\\0&1/(2\sigma ^{4})\end{pmatrix}}} Kullback–Leibler divergence 1 2 { (σ 0 σ 1) 2 + (μ 1 − μ 0) 2 σ 1 2 − 1 + ln σ 1 2 σ 0 2 } {\displaystyle {1 \over 2}\left\{\left({\frac {\sigma _{0}}{\sigma _{1}}}\right)^{2}+{\frac {(\mu _{1}-\mu _{0})^{2}}{\sigma _{1}^{2}}}-1+\ln {\sigma _{1}^{2} \over \sigma _{0}^{2}}\right\}} Expected shortfall μ + σ 1 2 π e − (q p (X − μ σ)) 2 2 1 − p {\displaystyle \mu +\sigma {\frac {{\frac {1}{\sqrt {2\pi }}}e^{\frac {-\left(q_{p}\left({\frac {X-\mu }{\sigma }}\right)\right)^{2}}{2}}}{1-p}}} [1] Part of a series on statisticsProbability theory Probability Axioms Determinism System Indeterminism Randomness Probability space Sample space Event Collectively exhaustive events Elementary event Mutual exclusivity Outcome Singleton Experiment Bernoulli trial Probability distribution Bernoulli distribution Binomial distribution Exponential distribution Normal distribution Pareto distribution Poisson distribution Probability measure Random variable Bernoulli process Continuous or discrete Expected value Variance Markov chain Observed value Random walk Stochastic process Complementary event Joint probability Marginal probability Conditional probability Independence Conditional independence Law of total probability Law of large numbers Bayes' theorem Boole's inequality Venn diagram Tree diagram vte In probability theory and statistics, a normal distribution or Gaussian distribution is a type of continuous probability distribution for a real-valued random variable. This distribution is symmetric around zero, unbounded at z = 0 {\textstyle z=0} , and has the characteristic function ϕ Z (t) = (1 + t 2) − 1 / 2 {\textstyle \phi _{Z}(t)=(1+t^{2})^{-1/2}} . If X 1 {\textstyle X_{1}} and X 2 {\textstyle X_{2}} are two independent normal random variables, with means μ 1 {\textstyle \mu _{1}} , μ 2 {\textstyle \mu _{2}} and variances σ 1 2 {\textstyle \sigma _{1}^{2}} , σ 2 2 {\textstyle \sigma _{2}^{2}} , then their sum X 1 + X 2 {\textstyle X_{1}+X_{2}} will also be normally distributed,[proof] with mean μ 1 + μ 2 {\textstyle \mu _{1}+\mu _{2}} and variance σ 1 2 + σ 2 2 {\textstyle \sigma _{1}^{2}+\sigma _{2}^{2}} . For b = ∞ {\textstyle b=\infty } this is known as the inverse Mills ratio. ^ Peirce, Charles S. The Kaniadakis κ-Gaussian distribution is a generalization of the Gaussian distribution which arises from the Kaniadakis statistics, being one of the Kaniadakis distributions. 102) ^ Lyon, A. Typically the null hypothesis H0 is that the observations are distributed normally with unspecified mean μ and variance σ2, versus the alternative Ha that the distribution is arbitrary. Kinderman, Albert J.; Monahan, John F. Each of the priors has a hyperparameter specifying the number of pseudo-observations, and in each case this controls the relative variance of that prior. ^ Why Most Published Research Findings Are False, John P. H. Shore, H (1982). {\displaystyle {\begin{aligned}\operatorname {E} \left[X^{p}\right]&=\sigma ^{p}\cdot {\left(-i{\sqrt {2}}\right)}^{p}\,U\left(-{\frac {p}{2}},{\frac {1}{2}},-{\frac {\mu ^{2}}{2\sigma ^{2}}}\right),\\\operatorname {E} \left[|X|^{p}\right]&=\sigma ^{p}\cdot 2^{p/2}{\frac {\Gamma \left({\frac {1+p}{2}}\right)}{\sqrt {\pi }}}\,{}_{1}F_{1}\left(-{\frac {p}{2}},{\frac {1}{2}},-{\frac {\mu ^{2}}{2\sigma ^{2}}}\right).\end{aligned}}} These expressions remain valid even if p {\displaystyle p} is not an integer. Handbook of mathematical functions with formulas, graphs, and mathematical tables, by Abramowitz, M.; and Stegun, I. {\displaystyle {\begin{aligned}p(\mu \mid \sigma ^{2};\mu _{0},n_{0})&\sim {\mathcal {N}}(\mu _{0},\sigma ^{2}/n_{0})={\frac {1}{\sqrt {2\pi {\frac {\sigma ^{2}}{n_{0}}}}}}\exp \left(-{\frac {n_{0}}{2\sigma ^{2}}}(\mu -\mu _{0})^{2}\right)\\&\propto (\sigma ^{2})^{-1/2}\exp \left(-{\frac {n_{0}}{2\sigma ^{2}}}(\mu -\mu _{0})^{2}\right)\\p(\sigma ^{2};\nu _{0},\sigma _{0}^{2})&\sim I\chi ^{2}(\nu _{0},\sigma _{0}^{2})=IG(\nu _{0}/2,\nu _{0}\sigma _{0}^{2}/2)\\&={\frac {(\sigma _{0}^{2}\nu _{0}/2)^{\nu _{0}/2}}{\Gamma (\nu _{0}/2)}}~{\frac {\exp \left[{\frac {-\nu _{0}\sigma _{0}^{2}}{2\sigma ^{2}}}\right]}{(\sigma ^{2})^{1+\nu _{0}/2}}}\\&\propto {(\sigma ^{2})}^{-(1+\nu _{0}/2)}\exp \left[{\frac {-\nu _{0}\sigma _{0}^{2}}{2\sigma ^{2}}}\right].\end{aligned}}} Therefore, the joint prior is p (μ , σ 2 ; μ 0 , n 0 , ν 0 , σ 0 2) = p (μ ∣ σ 2 ; μ 0 , n 0) p (σ 2 ; ν 0 , σ 0 2) ∝ (σ 2) − (ν 0 + 3) / 2 exp [− 1 2 σ 2 (ν 0 σ 0 2 + n 0 (μ − μ 0) 2)] . ISBN 978-0387-30303-1. Laplace, Pierre-Simon (1812). 19 (124): 19–32. Wikidata Q55897617. For example, to find the 95th percentile when the mean is 100 and the standard deviation is 20, you should have invNorm(0.95,100,20). Press [ENTER]. When the variance is unknown, analysis may be done directly in terms of the variance, or in terms of the precision, the reciprocal of the variance. 1258) ^ Patel & Read (1996, [2.1.8]) ^ Papoulis, Athanasios. Another approximation, somewhat less accurate, is the single-parameter approximation: z = − 0.4115 { 1 − p p + log [1 − p p] − 1 } , p ≥ 1 / 2 {\displaystyle z=-0.4115\left\{{\frac {1-p}{p}}+\log \left[{\frac {1-p}{p}}\right]-1\right\},\qquad p\geq 1/2} The latter had served to derive a simple approximation for the loss integral of the normal distribution, defined by L (z) = ∫ z ∞ (u − z) φ (u) d u = ∫ z ∞ [1 − Φ (u)] d u L (z) ≈ { 0.4115 (p 1 − p) − z , p < 1 / 2 , 0.4115 (1 − p p) , p ≥ 1 / 2. doi:10.1214/aoms/1177731647. This can be shown more easily by rewriting the variance as the precision, i.e. using τ = 1/σ2. Lexis, Wilhelm (1878). Shore, H (2012). Halperin, Max; Hartley, Herman O.; Hoel, Paul G. 42 (1): 3:1–14. 13 (4): 359–62. LCCN 65-12253. Infinity is just so big that by comparison, bigness itself looks really titchy. (2022). The GNU Scientific Library calculates values of the standard normal cumulative distribution function using Hart's algorithms and approximations with Chebyshev polynomials. {\displaystyle {\sqrt {n}}({\hat {\sigma }}^{2}-\sigma ^{2})\simeq {\sqrt {n}}(s^{2}-\sigma ^{2})\,\xrightarrow {d} \,{\mathcal {N}}(0,2\sigma ^{4}).} In particular, both estimators are asymptotically efficient for σ 2 {\textstyle \sigma ^{2}} . Of practical importance is the fact that the standard error of μ ^ {\displaystyle \textstyle {\hat {\mu }}} is proportional to 1 / n {\displaystyle \textstyle 1/{\sqrt {n}}} , that is, if one wishes to decrease the standard error by a factor of 10, one must increase the number of points in the sample by a factor of 100. Gauss Figure 6-10 For now, we will be working with the most common bell-shaped probability distribution known as the normal distribution, also called the Gaussian distribution, named after the German mathematician Johann Carl Friedrich Gauss. Statistics on the Table. The American Statistician. ed.). "V. A.: National Bureau of Standards. This text does not use probability tables and will instead rely on technology to compute the area under the curve. Testing Statistical Hypotheses (2nd ed.). {\textstyle \varphi '(x)=-x\varphi (x).} Its second derivative is φ ″ (x) = (x 2 − 1) φ (x) {\textstyle \varphi ''(x)=(x^{2}-1)\varphi (x)} More generally, its nth derivative is φ (n) (x) = (− 1) n He n (x) φ (x) , {\textstyle \varphi ^{(n)}(x)=(-1)^{n}\operatorname {He} _{n}(x)\varphi (x),} where He n (x) {\textstyle \operatorname {He} _{n}(x)} is the nth (probabilist) Hermite polynomial.[24] The probability that a normally distributed variable X {\displaystyle X} with known μ {\displaystyle \mu } and σ 2 {\textstyle \sigma ^{2}} is in a particular set, can be calculated by using the fact that the fraction Z = (X − μ) / σ {\textstyle Z=(X-\mu)/\sigma } has a standard normal distribution. Excel: z = NORM.S.INV(0.9066) = 1.32. A vector X ∈ Rk is multivariate-normally distributed if any linear combination of its components Σkj=1aj Xj has a (univariate) normal distribution. ^ Lehmann, E. Outliers appear as points that are far away from the overall pattern of the plot. This property is called infinite divisibility.[47] Conversely, if X 1 {\textstyle X_{1}} and X 2 {\textstyle X_{2}} are independent random variables and their sum X 1 + X 2 {\textstyle X_{1}+X_{2}} has a normal distribution, then both X 1 {\textstyle X_{1}} and X 2 {\textstyle X_{2}} must be normal deviates.[48] This result is known as Cramér's decomposition theorem, and is equivalent to saying that the convolution of two distributions is normal if and only if both are normal. or, equivalently, L (z) ≈ { 0.4115 { 1 − log [p 1 − p] } , p < 1 / 2 , 0.4115 1 − p p , p ≥ 1 / 2 . The algorithm proceeds as follows: Generate two independent uniform deviates U and V; Compute X = √8/e (V − 0.5)/U; Optional: if X2 ≤ 5 − 4e1/4U then accept X and terminate algorithm; Optional: if X2 ≥ 4e−1.35/U + 1.4 then reject X and start over from step 1; If X2 ≤ −4 lnU then accept X, otherwise start over the algorithm. Archived from the original (PDF) on February 29, 2012. doi:10.2307/2684031. If Z {\displaystyle Z} is a standard normal deviate, then X = σ Z + μ {\textstyle X=\sigma Z+\mu } will have a normal distribution with expected value μ {\displaystyle \mu } and standard deviation σ {\displaystyle \sigma } . For any non-negative integer p {\displaystyle p} , the plain central moments are:[25] E [(X − μ) p] = { 0 if p is odd, σ p (p − 1) ! ! if p is even. In Smith, David Eugene (ed.). Archived from the original (PDF) on March 31, 2023. Cover, Thomas M.; Thomas, Joy A. {\displaystyle \operatorname {erf} (x)={\frac {1}{\sqrt {\pi }}}\int _{-x}^{x}e^{-t^{2}}\,dt={\frac {2}{\sqrt {\pi }}}\int _{0}^{x}e^{-t^{2}}\,dt\,.} These integrals cannot be expressed in terms of elementary functions, and are often said to be special functions. JSTOR 2684031. The values Φ(x) may be approximated very accurately by a variety of methods, such as numerical integration, Taylor series, asymptotic series and continued fractions. 11 (4). It is roughly approximated to in certain distributions; for this reason, and on account for its beautiful simplicity, we may, perhaps, use it as a first approximation, particularly in theoretical investigations.—Pearson (1901) There are statistical methods to empirically test that assumption; see the above Normality tests section. The dual expectation parameters for normal distribution are η1 = μ and η2 = μ2 + σ2. Some authors prefer to instead work with the characteristic function E[eitX] = eiμt − σ2t2/2 and ln E[eitX] = iμt − 1/2σ2t2. Supplement to the Journal of the Royal Statistical Society 3 (2): 178–184 ^ Lukacs, Eugene (March 1942). The confidence interval for σ can be found by taking the square root of the interval bounds for σ2. (1968). The lower tail area for x1 would have 0.02/2 = 0.01. The absolute value of X {\displaystyle X} has folded normal distribution: | X | ∼ N f (μ , σ 2) {\textstyle {\left|X\right|\sim N_{f}(\mu ,\sigma ^{2})}} . {\displaystyle {\begin{aligned}p(\mu ,\sigma ^{2};\mu _{0},n_{0},\nu _{0},\sigma _{0}^{2})&=p(\mu \mid \sigma ^{2};\mu _{0},n_{0})\,p(\sigma ^{2};\nu _{0},\sigma _{0}^{2})\\&\propto (\sigma ^{2})^{-(\nu _{0}+3)/2}\exp \left[-{\frac {1}{2\sigma ^{2}}}\left(\nu _{0}\sigma _{0}^{2}+n_{0}(\mu -\mu _{0})^{2}\right)\right].\end{aligned}}} The likelihood function from the section above with known variance is: p (X ∣ μ , σ 2) = (1 2 π σ 2) n / 2 exp [− 1 2 σ 2 (∑ i = 1 n (x i − μ) 2)] {\displaystyle {\begin{aligned}p(\mathbf {X} \mid \mu ,\sigma ^{2})&=\left({\frac {1}{2\pi \sigma ^{2}}}\right)^{n/2}\exp \left[-{\frac {1}{2\sigma ^{2}}}\left(\sum _{i=1}^{n}(x_{i}-\mu)^{2}\right)\right]\end{aligned}}} Writing it in terms of variance rather than precision, we get: p (X ∣ μ , σ 2) = (1 2 π σ 2) n / 2 exp [− 1 2 σ 2 (∑ i = 1 n (x i − x ¯) 2 + n (x ¯ − μ) 2)] ∝ σ 2 − n / 2 exp [− 1 2 σ 2 (S + n (x ¯ − μ) 2)] {\displaystyle {\begin{aligned}p(\mathbf {X} \mid \mu ,\sigma ^{2})&=\left({\frac {1}{2\pi \sigma ^{2}}}\right)^{n/2}\exp \left[-{\frac {1}{2\sigma ^{2}}}\left(\sum _{i=1}^{n}(x_{i}-{\bar {x}})^{2}+n({\bar {x}}-\mu)^{2}\right)\right]\\&\propto {\sigma ^{2}}^{-n/2}\exp \left[-{\frac {1}{2\sigma ^{2}}}\left(S+n({\bar {x}}-\mu)^{2}\right)\right]\end{aligned}}} where S = ∑ i = 1 n (x i − x ¯) 2 . Figure 6-15 Compute the area under the standard normal distribution that is 2 standard deviations from the mean. For example, to find P(Z < -1.37) you should have normalcdf(-1E99,-1.37). (1969). Only in 3% of the cases, where the combination of those two falls outside the "core of the ziggurat" (a kind of rejection sampling using logarithms), do exponentials and more uniform random numbers have to be employed. ^ a b Krishnamoorthy (2006, p. It states that, under some conditions, the average of many samples (observations) of a random variable with finite mean and variance is itself a random variable—whose distribution converges to a normal distribution as the number of samples increases. McPherson, Glen (1990). (1965). Rohrbasser, Jean-Marc; Véron, Jacques (2003). LCCN 64-60036. The central limit theorem implies that those statistical parameters will have asymptotically normal distributions. "Stat260: Bayesian Modeling and Inference: The Conjugate Prior for the Normal Distribution" (PDF). December 5, 2007. doi:10.1016/j.jeconom.2008.12.014. "A Property of Normal Distribution". Théorie analytique des probabilités [Analytical theory of probabilities]. To find the area under the probability density curve involves calculus so we will need to rely on technology to find the area. doi:10.2307/2681417. 299 (1–2). Free Press. doi:10.1214/aos/1176348248. Variables such as SAT scores and heights of United States adults closely follow the normal distribution. ^ Sun, Jingchao; Kong, Maiying; Pal, Subhadip (June 22, 2021). In this regard a series of Hadamard transforms can be combined with random permutations to turn arbitrary data sets into a normally distributed data. Zelen, Marvin; Severo, Norman C. ^ Weisstein, Eric W. F. {\displaystyle \Phi '(x_{n})={\frac {1}{\sqrt {2\pi }}}e^{-x_{n}^{2}/2}\,.} When the repeated computations converge to an error below the chosen acceptably small value, x will be the value needed to obtain a Φ (x) {\textstyle \Phi (x)} of the desired value, Φ (desired) {\displaystyle \Phi ({\text{desired}})} . This fact is widely used in determining sample sizes for opinion polls and the number of trials in Monte Carlo simulations. Handbook of Statistical Distributions with Applications. "Normal Distribution Function". 148. 189) ^ Besides those specifically referenced here, such use is encountered in the works of Peirce, Galton (Galton (1889, chapter V)) and Lexis (Lexis (1878), Rohrbasser & Véron (2003)) c. ISBN 978-0-387-95036-5. Statistical Inference (2nd ed.). Then if x ∼ N (μ , 1 / τ) {\textstyle x\sim {\mathcal {N}}(\mu ,1/\tau)} and μ ∼ N (μ 0 , 1 / τ 0) , {\textstyle \mu \sim {\mathcal {N}}(\mu _{0},1/\tau _{0}),} we proceed as follows. This will return the percentile for the x value. The generalized normal distribution, also known as the exponential power distribution, allows for distribution tails with thicker or thinner asymptotic behaviors. Unimodal (one mode). This variate is also called the standardized form of X {\displaystyle X} . The ziggurat algorithm[63] is faster than the Box–Muller transform and still exact. Bryc, Wlodzimierz (1995). (1992). Normal distribution is a symmetric distribution. We could have also found the z-score that corresponds to the top 10%. A z-score is the number of standard deviations an observation x is above or below the mean μ. Logically, this originates as follows: From the analysis of the case with unknown mean but known variance, we see that the update equations involve sufficient statistics computed from the data consisting of the mean of the data points and the total variance of the data points, computed in turn from the known variance divided by the number of data points. Statistics and Public Policy. doi:10.1081/sta-200052102. These steps can be greatly improved[62] so that the logarithm is rarely evaluated. Their ratio follows the standard Cauchy distribution: X 1 / X 2 ∼ Cauchy (0 , 1) {\textstyle X_{1}/X_{2}\sim \operatorname {Cauchy} (0,1)} . The standard deviation of the distribution is σ {\displaystyle \sigma } (sigma). Its antiderivative (indefinite integral) can be expressed as follows: ∫ Φ (x) d x = x Φ (x) + φ (x) + C . In 1823 Gauss published his monograph "Theoria combinationis observationum erroribus minimis obnoxiae" where among other things he introduces several important statistical concepts, such as the method of least squares, the method of maximum likelihood, and the normal distribution. ISBN 978-0-471-58495-7. This will get you a menu of probability distributions. Thus, the normal distribution, while not perfect for any single problem, is very useful for a variety of problems. "Normal Product Distribution". Archived from the original (PDF) on March 7, 2016. 25 (2): 389–394. The exponential of X {\displaystyle X} is distributed log-normally: e X ∼ ln (N (μ , σ 2)) {\textstyle e^{X}\sim \ln(N(\mu ,\sigma ^{2}))} . Its first derivative is f ′ (x) = − x − μ σ 2 f (x) . {\displaystyle \operatorname {E} \left[(X-\mu)^{p}\right]={\begin{cases}0&{\text{if }}p{\text{ is odd,}}\\\sigma ^{p}(p-1)!!&{\text{if }}p{\text{ is even.}}\end{cases}}} Here n ! ! {\textstyle n!!} denotes the double factorial, that is, the product of all numbers from n {\displaystyle n} to 1 that have the same parity as n . Although Gauss was the first to suggest the normal distribution law, Laplace made significant contributions.[note 4] It was Laplace who first posed the problem of aggregating several observations in 1774,[74] although his own solution led to the Laplacian distribution. Note that there is no assumption of independence.[39] As the number of discrete events increases, the function begins to resemble a normal distribution. {\displaystyle s^{2}\sim {\frac {\sigma ^{2}}{n-1}}\cdot \chi _{n-1}^{2},\qquad {\hat {\sigma }}^{2}\sim {\frac {\sigma ^{2}}{n}}\cdot \chi _{n-1}^{2}.} The first of these expressions shows that the variance of s 2 {\textstyle s^{2}} is equal to 2 σ 4 / (n − 1) {\textstyle 2\sigma ^{4}/(n-1)} , which is slightly greater than the σσ-element of the inverse Fisher information matrix I − 1 {\displaystyle \textstyle {\mathcal {I}}^{-1}} , which is 2 σ 4 / n {\textstyle 2\sigma ^{4}/n} . In his notation φΔ is the probability density function of the measurement errors of magnitude Δ. {\displaystyle {\begin{aligned}\Phi ^{(0)}(x_{0})&={\frac {1}{\sqrt {2\pi }}}\int _{-\infty }^{x_{0}}e^{-t^{2}/2}\,dt\\\Phi ^{(1)}(x_{0})&={\frac {1}{\sqrt {2\pi }}}e^{-x_{0}^{2}/2}\\\Phi ^{(n)}(x_{0})&=-\left(x_{0}\Phi ^{(n-1)}(x_{0})+(n-2)\Phi ^{(n-2)}(x_{0})\right),&n\geq 2\,.\end{aligned}}} An application for the above Taylor series expansion is to use Newton's method to reverse the computation. In the following sections we look at some special cases. This implies that the estimator is finite-sample efficient. In addition, consider that the knowledge of the posterior comes from a combination of the knowledge of the prior and likelihood, so it makes sense that we are more certain of it than of either of its components.) The above formula reveals why it is more convenient to do Bayesian analysis of conjugate priors for the normal distribution in terms of the precision. Please help improve this article by adding citations to reliable sources. "Wilhelm Lexis: The Normal Length of Life as an Expression of the "Nature of Things"". {\textstyle U.} [26] E [X p] = σ p ⋅ (− i 2) p U (− p 2 , 1 2 , − μ 2 2 σ 2) , E [| X | p] = σ p ⋅ 2 p / 2 Γ (1 + p 2) π 1 F 1 (− p 2 , 1 2 , − μ 2 2 σ 2) . Note that as the distribution becomes closer to a normal distribution the dots on the quantile plot will be in a straighter line. The following table gives the quantile z p {\textstyle z_{p}} such that X {\displaystyle X} will lie in the range μ ± z p σ {\textstyle \mu \pm z_{p}\sigma } with a specified probability p {\displaystyle p} . ACM Transactions on Mathematical Software. Find P(X ≥ 69.5) where X ~ N(67, 2.5). ISBN 978-0-8218-2103-9. More precisely, the probability that a normal deviate lies in the range between μ − n σ {\textstyle \mu -n\sigma } and μ + n σ {\textstyle \mu +n\sigma } is given by F (μ + n σ) − F (μ − n σ) = Φ (n) − Φ (− n) = erf (n 2) . In addition, since x i x j = x j x i {\textstyle x_{i}x_{j}=x_{j}x_{i}} , only the sum a i j + a j i {\textstyle a_{ij}+a_{ji}} matters for any off-diagonal elements of A, and there is no loss of generality in assuming that A is symmetric. More specifically, where X 1 , … , X n {\textstyle X_{1},\ldots ,X_{n}} are independent and identically distributed random variables with the same arbitrary distribution, zero mean, and variance σ 2 {\textstyle \sigma ^{2}} and Z {\displaystyle Z} is their mean scaled by n {\textstyle {\sqrt {n}}} Z = n (1 n ∑ i = 1 n X i) {\displaystyle Z={\sqrt {n}}\left({\frac {1}{n}}\sum _{i=1}^{n}X_{i}\right)} Then, as n {\displaystyle n} increases, the probability distribution of Z {\displaystyle Z} will tend to the normal distribution with zero mean and variance σ 2 {\displaystyle \sigma ^{2}} . We'll explore this and other real-world uses of the normal distribution in this section. Their product Z = X 1 X 2 {\textstyle Z=X_{1}X_{2}} follows the product distribution[43] with density function f Z (z) = π − 1 K 0 (| z |) {\textstyle f_{Z}(z)=\pi ^{-1}K_{0}(|z|)} where K 0 {\textstyle K_{0}} is the modified Bessel function of the second kind. "The Q-function". ISBN 978-0-534-24312-8. Methods of Information Geometry. The value of zero is the median on a standard normal distribution, so 50% of the area lies to the left of z = 0. 6, paragraph 327. The square of X / σ {\textstyle X/\sigma } has the noncentral chi-squared distribution with one degree of freedom: X 2 / σ 2 ∼ χ 1 2 (μ 2 / σ 2) {\textstyle X^{2}/\sigma ^{2}\sim \chi _{1}^{2}(\mu ^{2}/\sigma ^{2})} . Excel: P(-1.37 ≤ Z ≤ 1.68) =NORM.S.DIST(1.68,TRUE)-NORM.S.DIST(-1.37,TRUE) = 0.8682. ^ Basu, D.; Laha, R. Proof The prior distributions are p (μ ∣ σ 2 ; μ 0 , n 0) ∼ N (μ 0 , σ 2 / n 0) = 1 2 π σ 2 n 0 exp (− n 0 2 σ 2 (μ − μ 0) 2) ∝ (σ 2) − 1 / 2 exp (− n 0 2 σ 2 (μ − μ 0) 2) p (σ 2 ; ν 0 , σ 0 2) ∼ I χ 2 (ν 0 , σ 0 2) = I G (ν 0 / 2 , ν 0 σ 0 2 / 2) = (σ 0 2 ν 0 / 2) ν 0 / 2 Γ (ν 0 / 2) exp [− ν 0 σ 0 2 2 σ 2] (σ 2) 1 + ν 0 / 2 ∝ (σ 2) − (1 + ν 0 / 2) exp [− ν 0 σ 0 2 2 σ 2] . 19 (3): 12–14. The normal distribution is often referred to as N (μ , σ 2) {\textstyle N(\mu ,\sigma ^{2})} or N (μ , σ 2) {\displaystyle {\mathcal {N}}(\mu ,\sigma ^{2})} .[11] Thus when a random variable X {\displaystyle X} is normally distributed with mean μ {\displaystyle \mu } and standard deviation σ {\displaystyle \sigma } , one may write X ∼ N (μ , σ 2) . Sankhyā. For example, the SAT's traditional range of 200–800 is based on a normal distribution with a mean of 500 and a standard deviation of 100. Keep in mind that the posterior update values serve as the prior distribution when further data is handled. MR 0006626. In Excel we can use the following formula =NORM.DIST(600,501,116,TRUE)- NORM.DIST(400,501,116,TRUE). "Recommended Standards for Statistical Symbols and Notation. Different approximations are used depending on the desired level of accuracy. {\displaystyle \varphi _{X}(t)={\hat {f}}(-t)\,.} The moment generating function of a real random variable X {\displaystyle X} is the expected value of e t X {\textstyle e^{tX}} , as a function of the real parameter t {\displaystyle t} . Further information: Quantile function § Normal distribution The quantile function of a distribution is the inverse of the cumulative distribution function. p. ISBN 978-0-88275-642-4. ^ Harris, Frank E. This is equivalent to saying that the standard normal distribution Z {\displaystyle Z} can be scaled/stretched by a factor of σ {\displaystyle \sigma } and shifted by μ {\displaystyle \mu } to yield a different normal distribution, called X {\displaystyle X} . Figure 6-14 TI Calculator: P(-1.37 ≤ Z ≤ 1.68) = normalcdf(-1.37,1.68,0,1) = 0.8682. Stigler in Statistical Science 1 (3), 1986: JSTOR 2245476. The rainfall data are represented by plotting positions as part of the cumulative frequency analysis. Gauss himself apparently coined the term with reference to the "normal equations" involved in its applications, with normal having its technical meaning of orthogonal rather than usual.[81] However, by the end of the 19th century some authors[note 5] had started using the name normal distribution, where the word "normal" was used as an adjective – the term now being seen as a reflection of the fact that this distribution was seen as typical, common – and thus normal. For any non-negative integer p , {\textstyle p,} E [| X − μ | p] = σ p (p − 1) ! ! ⋅ { 2 π if p is odd 1 if p is even = σ p ⋅ 2 p / 2 Γ (p + 1 2) π . a (x − y) 2 + b (x − z) 2 = (a + b) (x − a y + b z a + b) 2 + a b a + b (y − z) 2 {\displaystyle a(x-y)^{2}+b(x-z)^{2}=(a+b)\left(x-{\frac {ay+bz}{a+b}}\right)^{2}+{\frac {ab}{a+b}}(y-z)^{2}} This equation rewrites the sum of two quadratics in x by expanding the squares, grouping the terms in x, and completing the square. In Excel you would have to do this in two separate cells =NORM.INV(0.01,246300,15000) = $211,404.78 and =NORM.INV(0.99,246300,15000) = $281,195.22. This is not the case, however, with the total variance of the mean: As the unknown variance increases, the total variance of the mean will increase proportionately, and we would like to capture this dependence. Annals of Mathematical Statistics. Enter the area to the left of the x value, μ, and σ with a comma between each. This distribution is different from the Gaussian q-distribution above. Oxford University Press. 6. Some mathematicians such as Benoit Mandelbrot have argued that log-Levy distributions, which possesses heavy tails would be a more appropriate model, in particular for the analysis for stock market crashes. {\textstyle x=\mu +\sigma .} [22] Its density is log-concave.[22] Its density is infinitely differentiable, indeed supersmooth of order 2.[23] Furthermore, the density φ {\displaystyle \varphi } of the standard normal distribution (i.e. μ = 0 {\textstyle \mu =0} and σ = 1 {\textstyle \sigma =1}) also has the following properties: Its first derivative is φ ′ (x) = − x φ (x) . wolfram.com. Marcel Dekker, Inc. However, many numerical approximations are known; see below for more. ^ Smith, José M. "Accuracy in random number generation". Gaussian q-distribution is an abstract mathematical construction that represents a q-analogue of the normal distribution. {\displaystyle {\begin{aligned}p(\mu ,\sigma ^{2}\mid \mathbf {X})&\propto p(\mu ,\sigma ^{2})\,p(\mathbf {X} \mid \mu ,\sigma ^{2})\\&\propto (\sigma ^{2})^{-(\nu _{0}+3)/2}\exp \left[-{\frac {1}{2\sigma ^{2}}}\left(\nu _{0}\sigma _{0}^{2}+n_{0}(\mu -\mu _{0})^{2}\right)\right]{\sigma ^{2}}^{-n/2}\exp \left[-{\frac {1}{2\sigma ^{2}}}\left(S+n({\bar {x}}-\mu)^{2}\right)\right]\\&=(\sigma ^{2})^{-(\nu _{0}+n+3)/2}\exp \left[-{\frac {1}{2\sigma ^{2}}}\left(\nu _{0}\sigma _{0}^{2}+S+n_{0}(\mu -\mu _{0})^{2}+n({\bar {x}}-\mu)^{2}\right)\right]\\&=(\sigma ^{2})^{-(\nu _{0}+n+3)/2}\exp \left[-{\frac {1}{2\sigma ^{2}}}\left(\nu _{0}\sigma _{0}^{2}+S+{\frac {n_{0}n}{n_{0}+n}}(\mu _{0}-{\bar {x}})^{2}+(n_{0}+n)\left(\mu -{\frac {n_{0}\mu _{0}+n{\bar {x}}}{n_{0}+n}}\right)^{2}\right)\right]\\&\propto (\sigma ^{2})^{-1/2}\exp \left[-{\frac {n_{0}+n}{2\sigma ^{2}}}\left(\mu -{\frac {n_{0}\mu _{0}+n{\bar {x}}}{n_{0}+n}}\right)^{2}\right]\\&\quad \times (\sigma ^{2})^{-(\nu _{0}/2+n/2+1)}\exp \left[-{\frac {1}{2\sigma ^{2}}}\left(\nu _{0}\sigma _{0}^{2}+S+{\frac {n_{0}n}{n_{0}+n}}(\mu _{0}-{\bar {x}})^{2}\right)\right]\\&={\mathcal {N}}_{\mu \mid \sigma ^{2}}\left({\frac {n_{0}\mu _{0}+n{\bar {x}}}{n_{0}+n}},{\frac {\sigma ^{2}}{n_{0}+n}}\right)\cdot {\rm {IG}}_{\sigma ^{2}}\left({\frac {1}{2}}(\nu _{0}+n),{\frac {1}{2}}\left(\nu _{0}\sigma _{0}^{2}+S+{\frac {n_{0}n}{n_{0}+n}}(\mu _{0}-{\bar {x}})^{2}\right)\right).\end{aligned}}} In other words, the posterior distribution has the form of a product of a normal distribution over p (μ ∣ σ 2) {\textstyle p(\mu |\sigma ^{2})} times an inverse gamma distribution over p (σ 2) {\textstyle p(\sigma ^{2})} , with parameters that are the same as the update equations above. 13 (1): 91–93. www.mathsisfun.com. Wrapped normal distribution – the Normal distribution applied to a circular domain Z-test – using the normal distribution ^ For example, this algorithm is given in the article Bc programming language. Anytime you are asked to find a probability of Z use the standard normal distribution. Such variables may be better described by other distributions, such as the log-normal distribution or the Pareto distribution. ^ Oosterbaan, Roland J. ISBN 978-0-8247-9342-5. The 25th percentile would have to be below the mean since the mean = median = 50th percentile for a bell-shaped distribution. x 0 {\textstyle x_{0}} may be a value from a distribution table, or an intelligent estimate followed by a computation of Φ (x 0) {\textstyle \Phi (x_{0})} using any desired means to compute. The upper value of x2 will have a left tail area of 0.99. {\displaystyle {\begin{aligned}\operatorname {E} \left[|X-\mu |^{p}\right]&=\sigma ^{p}(p-1)!!\cdot {\begin{cases}{\sqrt {\frac {2}{\pi }}}&{\text{if }}p{\text{ is odd}}\\1&{\text{if }}p{\text{ is even}}\end{cases}}\\&=\sigma ^{p}\cdot {\frac {2^{p/2}\Gamma \left({\frac {p+1}{2}}\right)}{\sqrt {\pi }}}.\end{aligned}}} The last formula is valid also for any non-integer p > − 1. The History of Statistics: The Measurement of Uncertainty before 1900. Newton's method is ideal to solve this problem because the first derivative of Φ (x) {\textstyle \Phi (x)} , which is an integral of the normal standard distribution, is the normal standard distribution, and is readily available to use in the Newton's method solution. Figure 6-21 Figure 6-22 Figure 6-23 Figure 6-24 Sometimes you will be given an area or probability and have to find the associated random variable x or z-score. ^ Park, Sung Y.; Bera, Anil K. ISBN 978-0-674-83601-3. CiteSeerX 10.1.1.511.9750. In biology, the logarithm of various variables tend to have a normal distribution, that is, they tend to have a log-normal distribution (after separation on male/female subpopulations), with examples including: Measures of size of living tissue (length, height, skin area, weight);[55] The length of inert appendages (hair, claws, nails, teeth) of biological specimens, in the direction of growth; presumably the thickness of tree bark also falls under this category; Certain physiological measurements, such as blood pressure of adult humans. Prior to the handheld calculators and personal computers, there were probability tables made to look up these areas. JSTOR 2347972. The two estimators are also both asymptotically normal: n (σ ^ 2 − σ 2) ≃ n (s 2 − σ 2) → d N (0 , 2 σ 4) . Mood (1950) Introduction to the Theory of Statistics.[84] Mathematics portal Bates distribution – similar to the Irwin–Hall distribution, but rescaled back into the 0 to 1 range Behrens–Fisher problem – the long-standing problem of testing whether two normal samples with different variances have same means; Bhattacharyya distance – method used to separate mixtures of normal distributions Erdős–Kac theorem – on the occurrence of the normal distribution in number theory Full width at half maximum Gaussian blur – convolution, which uses the normal distribution as a kernel Gaussian function Modified half-normal distribution[85] with the pdf on (0 , ∞) {\textstyle (0,\infty)} is given as f (x) = 2 β α 2 x α − 1 exp (− β x 2 + γ x) Ψ (α 2 , γ β) {\textstyle f(x)={\frac {2\beta ^{\frac {\alpha }{2}}x^{\alpha -1}\exp(-\beta x^{2}+\gamma x)}{\Psi {\left({\frac {\alpha }{2}},{\frac {\gamma }{\sqrt {\beta }}}\right)}}}} , where Ψ (α , z) = 1 Ψ 1 ((α , 1 2) (1 , 0) ; z) {\textstyle \Psi (\alpha ,z)={}_{1}\Psi _{1}\left({\begin{matrix}\left(\alpha ,{\frac {1}{2}}\right)\\(1,0)\end{matrix}};z\right)} denotes the Fox–Wright Psi function. {\displaystyle t={\frac {{\overline {X}}-\mu }{S/{\sqrt {n}}}}={\frac {{\frac {1}{n}}(X_{1}+\cdots +X_{n})-\mu }{\sqrt {{\frac {1}{n(n-1)}}\left[(X_{1}-{\overline {X}})^{2}+\cdots +(X_{n}-{\overline {X}})^{2}\right]}}}\sim t_{n-1}.} If X 1 , X 2 , … , X n {\textstyle X_{1},X_{2},\ldots ,X_{n}} , Y 1 , Y 2 , … , Y m {\textstyle Y_{1},Y_{2},\ldots ,Y_{m}} are independent standard normal random variables, then the ratio of their normalized sums of squares will have the F-distribution with (n, m) degrees of freedom:[46] F = (X 1 2 + X 2 2 + ⋯ + X n 2) / n (Y 1 2 + Y 2 2 + ⋯ + Y m 2) / m ∼ F n , m . ISBN 978-1-58488-635-8. G. ^ Bryc (1995, p. A random variable with a Gaussian distribution is said to be normally distributed, and is called a normal deviate. 249. English translation. Mark 1.39 on the number line and shade to the left of z = 1.39. 144) ^ Stigler (1978, p. It is one of the few distributions that are stable and that have probability density functions that can be expressed analytically, the others being the Cauchy distribution and the Lévy distribution. Order Non-central moment, E [X p] {\displaystyle \operatorname {E} \left[X^{p}\right]} Central moment, E [(X − μ) p] {\displaystyle \operatorname {E} \left[(X-\mu)^{p}\right]} 1 μ {\displaystyle \mu } 0 {\displaystyle 0} 2 μ 2 + σ 2 {\textstyle \mu ^{2}+\sigma ^{2}} σ 2 {\textstyle \sigma ^{2}} 3 μ 3 + 3 μ σ 2 {\textstyle \mu ^{3}+3\mu \sigma ^{2}} 0 {\displaystyle 0} 4 μ 4 + 6 μ 2 σ 2 + 3 σ 4 {\textstyle \mu ^{4}+6\mu ^{2}\sigma ^{2}+3\sigma ^{4}} 3 σ 4 {\textstyle 3\sigma ^{4}} 5 μ 5 + 10 μ 3 σ 2 + 15 μ σ 4 {\textstyle \mu ^{5}+10\mu ^{3}\sigma ^{2}+15\mu \sigma ^{4}} 0 {\displaystyle 0} 6 μ 6 + 15 μ 4 σ 2 + 45 μ 2 σ 4 + 15 σ 6 {\textstyle \mu ^{6}+15\mu ^{4}\sigma ^{2}+45\mu ^{2}\sigma ^{4}+15\sigma ^{6}} 15 σ 6 {\textstyle 15\sigma ^{6}} 7 μ 7 + 21 μ 5 σ 2 + 105 μ 3 σ 4 + 105 μ σ 6 {\textstyle \mu ^{7}+21\mu ^{5}\sigma ^{2}+105\mu ^{3}\sigma ^{4}+105\mu \sigma ^{6}} 0 {\displaystyle 0} 8 μ 8 + 28 μ 6 σ 2 + 210 μ 4 σ 4 + 420 μ 2 σ 6 + 105 σ 8 {\textstyle \mu ^{8}+28\mu ^{6}\sigma ^{2}+210\mu ^{4}\sigma ^{4}+420\mu ^{2}\sigma ^{6}+105\sigma ^{8}} 105 σ 8 {\textstyle 105\sigma ^{8}} The expectation of X {\displaystyle X} conditioned on the event that X {\displaystyle X} lies in an interval [a , b] {\textstyle [a,b]} is given by E [X ∣ a < X < b] = μ − σ 2 f (b) − f (a) F (b) − F (a) , {\displaystyle \operatorname {E} \left[X\mid a

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